Trigonometry

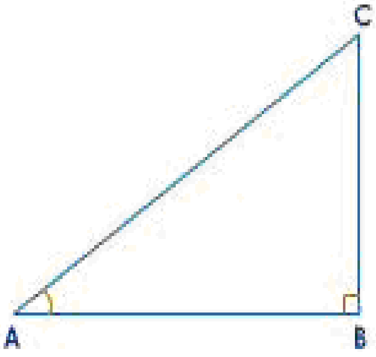
Introduction

Trigonometry is based on the right triangle. It is the study of the relationship between the sides and angles of a triangle. The height of a building, tree, tower, width of a river etc. can be determined with the help of trigonometry.

Trigonometric Ratios:

There are six t-ratios.

Δ ABC is a right-angled triangle, $\angle B = 90^0$.



(i) $\text{sine } A = \dfrac{\text{side opposite to} \angle A}{\text{hypotenuse}}$

$\text{sine } A = \dfrac{BC}{AC}$

In short, $\sin A = \dfrac{BC}{AC}\left[\dfrac{\text{opp.}}{\text{hyp.}}\right]$

(ii) $\text{cosine } A = \dfrac{\text{side adjacent to } \angle A}{\text{hypotenuse}}$

$\text{cosine } A = \dfrac{AB}{AC}$ In short, $\cos A = \dfrac{AB}{AC}\left[\dfrac{\text{adj.}}{\text{hyp.}}\right]$

(iii) $\text{tangent } A = \dfrac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A}$

$\text{tangent } A = \dfrac{BC}{AB}$ In short, $\tan A = \dfrac{BC}{AB}\left[\dfrac{\text{opp.}}{\text{adj.}}\right]$

Reciprocal Ratios:

(iv) cosecant $A = \frac{1}{\text{sine } A}$ In short, $\text{cosec } A = \frac{1}{\sin A}$

(v) secant $A = \frac{1}{\text{cosine } A}$ In short, $\sec A = \frac{1}{\cos A}$

(vi) cotangent $A = \frac{1}{\text{tangent } A}$ In short, $\cot A = \frac{1}{\tan A}$

$\sin C = \frac{AB}{AC}$

$\Rightarrow \text{cosec } C = \frac{AC}{AB}$

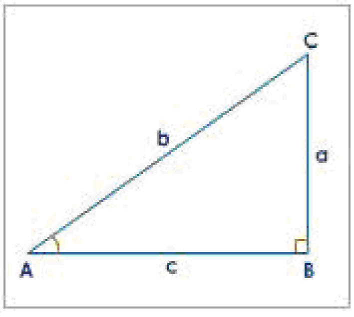
$\cos C = \frac{BC}{AC}$

$\Rightarrow \sec C = \frac{AC}{BC}$

$\tan C = \frac{AB}{BC}$

$\cot C = \frac{BC}{AB}$

Trigonometric Identities:



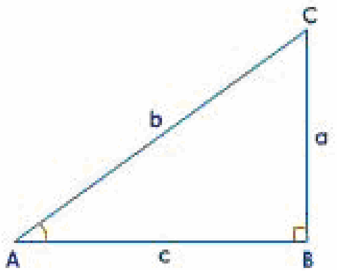
1. $\sin^2 A + \cos^2 A = 1$

2. $1 + \tan^2 A = \sec^2 A$

3. $1 + \cot^2 A = \text{cosec}^2 A$

Proof 1:

$a^2 + c^2 = b^2$ (by Pythagoras Theorem)



$\div$ by b^2 $\dfrac{a^2}{b^2} + \dfrac{c^2}{b^2} = \dfrac{b^2}{b^2}$

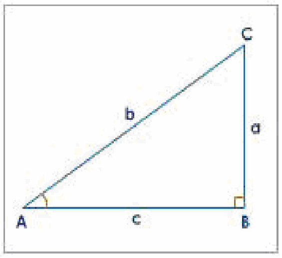
$\Rightarrow \left(\dfrac{a}{b}\right)^2 + \left(\dfrac{c}{b}\right)^2 = 1$

But $\sin A = \dfrac{a}{b}$ and $\cos A = \dfrac{c}{b}$,

$\therefore (\sin A)^2 + (\cos A)^2 = 1$

$\Rightarrow \sin^2 A + \cos^2 A = 1$

Proof 2:



$a^2 + c^2 = b^2$ (by Pythagoras Theorem)

$\div$ by c^2 $\dfrac{a^2}{c^2} + \dfrac{c^2}{c^2} = \dfrac{b^2}{c^2}$

$\Rightarrow \left(\dfrac{a}{c}\right)^2 + 1 = \left(\dfrac{b}{c}\right)^2$

But $\tan A = \dfrac{a}{c}$ and $\sec A = \dfrac{b}{c}$

$\therefore (\tan A)^2 + 1 = (\sec A)^2$

$\Rightarrow 1 + \tan^2 A = \sec^2 A$

Proof 3:

Again $a^2 + c^2 = b^2$ (by Pythagoras Theorem)

$$\div \text{ by } a^2 \quad \frac{a^2}{a^2} + \frac{c^2}{a^2} = \frac{b^2}{a^2}$$

$$\Rightarrow 1 + \left(\frac{c}{a}\right)^2 = \left(\frac{b}{a}\right)^2$$

But $\cot A = \frac{c}{a}$ and $\text{cosec } A = \frac{b}{a}$

$$\therefore 1 + (\cot A)^2 = (\text{cosec } A)^2$$

$$\Rightarrow 1 + \cot^2 A = \text{cosec}^2 A$$

Trigonometric Ratios of Standard Angles

0^0, 30^0, 45^0, 60^0 and 90^0 are called standard angles.

Trigonometric ratios of 45^0

Let $\angle A = 45^0, \angle C = 45^0, \angle B = 90^0$

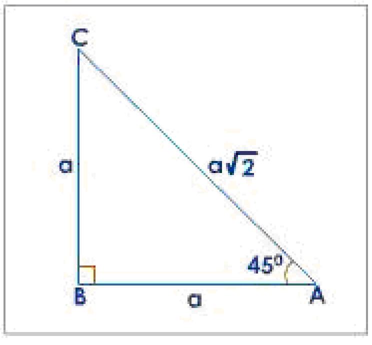
$$\therefore \angle A = \angle C$$

$$\therefore AB = BC = a$$

Using Pythagoras Theorem

$$AC^2 = a^2 + a^2 = 2a^2$$

$$AC = a\sqrt{2}$$



$$\sin 45^0 = \frac{\text{opposite side}}{\text{hypotensue}}$$

$= \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\cos 45^0 = \frac{\text{adjacent side}}{\text{hypotensue}}$

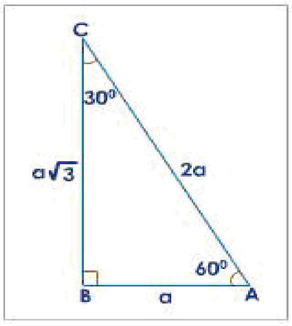
$= \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\tan 45^0 = \frac{\text{oppsotieside}}{\text{adjacent side}}$

$= \frac{a}{a} = 1$

Trigonometric ratios of 30^0 and 60^0

Let $\angle A = 60^0$ and $\angle C = 30^0$



In a $30^0 - 60^0 - 90^0$ triangle, it can be proved that the hypotenuse is double the side opposite to 30^0.

AC = 2AB

Let AB = a

$\therefore$ AC = 2a

Using Pythagoras Theorem

$BC^2 = (2a)^2 - a^2$

$= 4a^2 - a^2$

$= 3a^2$

$\therefore Bc = a\sqrt{3}$

$\sin 30^0 = \frac{a}{2a} = \frac{1}{2}$,

$\sin 60^0 = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$,

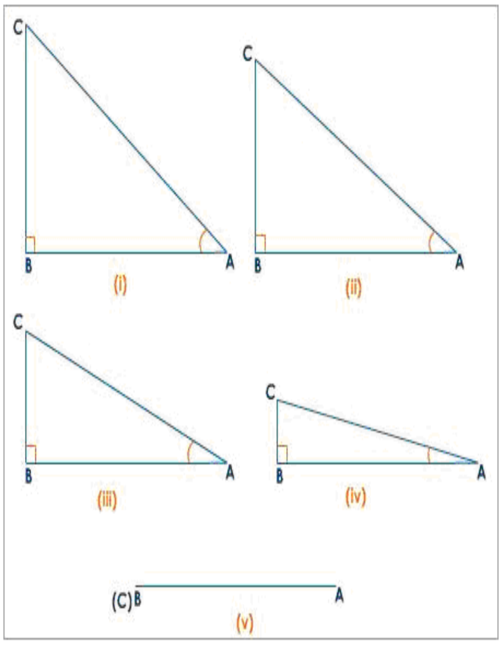
$$\cos 30^0 = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2},$$

$$\cos 60^0 = \frac{a}{2a} = \frac{1}{2},$$

$$\tan 30^0 = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}},$$

$$\tan 60^0 = \frac{a\sqrt{3}}{a} = \sqrt{3}$$

Trigonometric ratios of 0^0



ABC is a right-angled triangle with $\angle B = 90^0$. $\angle A$ is an acute angle and is made smaller and smaller till it becomes zero (fig v). As $\angle A$ gets smaller and smaller, the length of the side BC keeps decreasing. The point 'C' gets closer to the point B and at one time it coincides with 'B', so that $\angle A$ becomes zero.

If $\angle A = 0$, BC = 0

$$\sin A = \frac{BC}{AC} = \frac{0}{AC} = 0$$

$\Rightarrow \sin 0 = 0$

$$\cos A = \frac{AB}{AC} = \frac{AB}{AB} = 1$$

$\Rightarrow \cos 0 = 1$

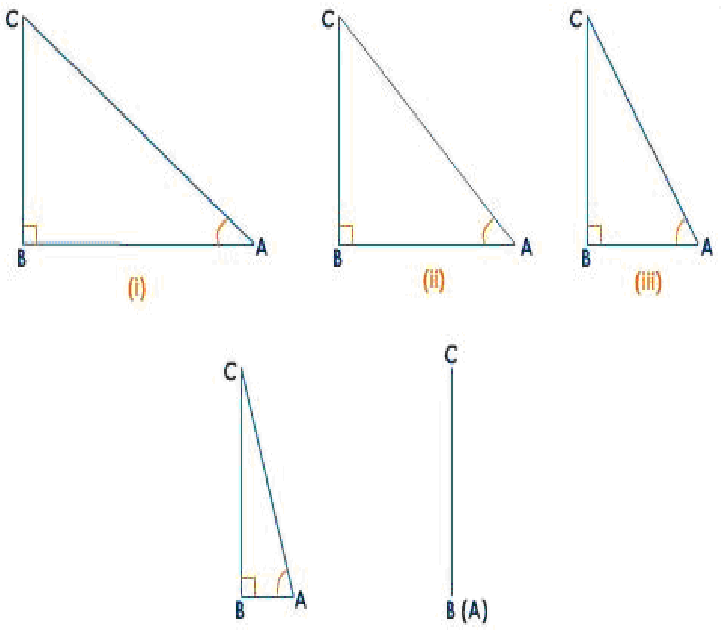
$$\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$$

$$\cot 0 = \frac{1}{\tan 0} = \frac{1}{0} = \text{not defined}$$

$$\sec 0 = \frac{1}{\cos 0} = \frac{1}{1} = 1$$

and $\operatorname{cosec} 0 = \frac{1}{\sin 0} = \frac{1}{0} = \text{not defined}$

Trigonometric ratios of 90^0



In right-angled triangle ABC, ∠A is made larger and larger till it becomes 90^0 (fig v). As ∠A gets larger and larger, ∠C gets smaller and smaller and the length of the side AB keeps decreasing. The point 'A' gets closer to the point B and at one time it coincides with 'B', so that ∠C becomes zero.

BC = AC

$$\text{Sin } 90 = \frac{BC}{AC} = \frac{AC}{AC} = 1$$

$$\text{Cos } 90 = \frac{AB}{AC} = \frac{0}{BC} = 0$$

$$\text{Tan } 90 = \frac{\sin 90}{\cos 90} = \frac{1}{0}, \text{ not defined}$$

$$\text{Cot } 90 = \frac{1}{\tan 90} = \frac{0}{1} = 0$$

$\text{Sec } 90 = \frac{1}{\cos 90} = \frac{1}{0}$ not defined

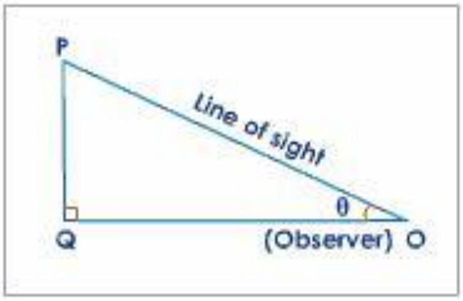
$\text{Cosec } 90 = \frac{1}{\sin 90} = \frac{1}{1} = 1$

Line of Sight

- It is an imaginary line drawn from the eye of the observer to the point of the object viewed by the observer.

Angle of Elevation

- It is the angle formed by the line of sight with the horizontal when the object is above the horizontal level. It is the case when we look up to see the object.



Angle of Depression

- It is the angle formed by the line of sight with the horizontal when the object is below the horizontal level. It is the case when we look down to see the object.